

Monday 24 June 2019 – Morning A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

• a scientific or graphical calculator



- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive no marks unless you show sufficient detail
 of the working to indicate that a correct method is used. You should communicate your
 method with correct reasoning.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.



Answer all the questions.

- 1 The matrix **A** is $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.
 - (a) Given that A represents a reflection, write down the eigenvalues of A. [1]
 - (b) Hence find the eigenvectors of **A**. [3]
 - (c) Write down the equation of the mirror line of the reflection represented by A. [1]
- 2 A surface S is defined by $z = 4x^2 + 4y^2 4x + 8y + 11$.
 - (a) Show that the point P(0.5, -1, 6) is the only stationary point on S. [2]
 - (b) (i) On the axes in the Printed Answer Booklet, draw a sketch of the contour of the surface corresponding to z = 42. [2]
 - (ii) By using the sketch in part (b)(i), deduce that P must be a minimum point on S. [3]
 - (c) In the section of S corresponding to y = c, the minimum value of z occurs at the point where x = a and z = 22.

Find all possible values of
$$a$$
 and c . [4]

3 The matrix **A** is $\begin{pmatrix} -1 & 2 & 4 \\ 0 & -1 & -25 \\ -3 & 5 & -1 \end{pmatrix}$.

Use the Cayley-Hamilton theorem to find \mathbf{A}^{-1} . [8]

- 4 *T* is the set $\{1, 2, 3, 4\}$. A binary operation is defined on *T* such that $a \bullet a = 2$ for all $a \in T$. It is given that (T, \bullet) is a group.
 - (a) Deduce the identity element in T, giving a reason for your answer. [2]
 - (b) Find the value of 1 3, showing how the result is obtained. [3]
 - (c) (i) Complete a group table for (T, \bullet) . [2]
 - (ii) State with a reason whether the group is abelian. [1]

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- 5 A financial institution models the repayment of a loan to a client in the following way.
 - An amount, £C, is loaned to the client at the start of the repayment period.
 - The amount owed n years after the start of the repayment period is $\pounds L_n$, so that $L_0 = C$.
 - At the end of each year, interest of $\alpha\%$ ($\alpha > 0$) of the amount owed at the start of that year is added to the amount owed.
 - Immediately after interest has been added to the amount owed a repayment of £R is made by the client
 - ullet Once L_n becomes negative the repayment is finished and the overpayment is refunded to the client.
 - (a) Show that during the repayment period, $L_{n+1} = aL_n + b$, giving a and b in terms of α and R. [2]
 - (b) Find the solution of the recurrence relation $L_{n+1} = aL_n + b$ with $L_0 = C$, giving your solution in terms of a, b, C and n. [5]
 - (c) Deduce from parts (a) and (b) that, for the repayment scheme to terminate, $R > \frac{\alpha C}{100}$. [2]

A client takes out a £30 000 loan at 8% interest and agrees to repay £3000 at the end of each year.

- (d) (i) Use an algebraic method to find the number of years it will take for the loan to be repaid.

 [3]
 - (ii) Taking into account the refund of overpayment, find the total amount that the client repays over the lifetime of the loan. [3]
- 6 (a) Given that $\sqrt{7}$ is an irrational number, prove that $a^2 7b^2 \neq 0$ for all $a, b \in \mathbb{Q}$ where a and b are not both 0. [2]
 - (b) A set G is defined by $G = \{a + b\sqrt{7} : a, b \in \mathbb{Q}, a \text{ and } b \text{ not both } 0\}$. Prove that G is a group under multiplication. (You may assume that multiplication is associative.)
 - (c) A subset H of G is defined by $H = \{1 + c\sqrt{7} : c \in \mathbb{Q}\}$. Determine whether or not H is a subgroup of (G, \times) .
 - (d) Using (G, \times) , prove by counter-example that the statement 'An infinite group cannot have a non-trivial subgroup of finite order' is false. [2]

END OF QUESTION PAPER

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